Effective equations for photonic-crystal waveguides and circuits

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We suggest a novel conceptual approach to describing the properties of waveguides and circuits in photonic crystals, based on effective discrete equations that include long-range interaction effects. We demonstrate, through the example of sharp waveguide bends, that our approach is very effective and accurate for the study of bound states and transmission spectra of photonic-crystal circuits and disclose the importance of evanescent modes in these phenomena. © 2002 Optical Society of America

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One of the most promising applications of photonic crystals is the possibility of creating compact integrated optical devices[1,2] that would be analogous to integrated circuits in electronics but that would operate entirely with light.

Usually, the properties of photonic crystals and photonic-crystal waveguides are studied by solution of Maxwell's equations numerically, and such calculations are time consuming. Moreover, the numerical approach does not always provide good physical insight. The purpose of this Letter is to suggest a novel approach, based on effective discrete equations, to describing many of the properties of photonic-crystal waveguides and circuits, including the example of the transmission spectra of sharp waveguide bends. The effective discrete equations that we derive below are somewhat analogous to the Kirchhoff equations for electric circuits. However, unlike in electronics, in photonic crystals both diffraction and interference become important, and thus the resulting equations involve long-range interaction effects.

To introduce our approach, we consider a two-dimensional (2D) photonic crystal consisting of infinitely long dielectric rods arranged in the form of a square lattice with lattice spacing a. We study the light propagation in the plane normal to the rods, assuming that the rods have a radius $r_0 = 0.18a$ and the dielectric constant $\epsilon_0 = 11.56$ (this corresponds to GaAs or Si at the wavelength $\sim 1.55\ \mu$m). For the electric field $E(\mathbf{x}, t) = \exp(-i\omega t)E(\mathbf{x}|\omega)$ polarized parallel to the rods, Maxwell's equations reduce to the eigenvalue problem

$$[\nabla^2 + (\omega/c)^2\epsilon(\mathbf{x})]E(\mathbf{x}|\omega) = 0\,, \tag{1}$$

which can be solved by the plane-wave method.[3] A perfect photonic crystal of this type possesses a large (38%) complete bandgap (from $\omega = 0.303 \times 2\pi c/a$ to $\omega = 0.444 \times 2\pi c/a$), and such crystals have been employed extensively during the past few years for the study of bound states in waveguides and bends,[4] transmission of light through sharp bends,[5,6] branches[7] and channel drop filters,[8] nonlinear localized modes in straight waveguides,[9] and perfect photonic crystals.[10] Recently, this type of photonic crystal with a 90°-bent waveguide was fabricated experimentally in macroporous silicon with $a = 0.57\ \mu$m and a complete bandgap at 1.55 μm.[11]

To create a waveguide circuit, we introduce a system of defects and assume, for simplicity, that the defects are identical rods of radius r_d (with ϵ_d) located at points $\mathbf{x}_m$, where m is the index number of the defect rods. In a photonic crystal with defects, the dielectric constant $\epsilon(\mathbf{x})$ can be presented as a sum of the periodic and defect-induced terms, i.e., $\epsilon(\mathbf{x}) = \epsilon_p(\mathbf{x}) + \epsilon_d(\mathbf{x})$, and therefore Eq. (1) can be written in an integral form:

$$E(\mathbf{x}|\omega) = (\omega/c)^2 \int \mathrm{d}^2\mathbf{y} G(\mathbf{x}, \mathbf{y}|\omega)\epsilon_d(\mathbf{y})E(\mathbf{y}|\omega)\,, \tag{2}$$

where $G(\mathbf{x}, \mathbf{y}|\omega)$ is the Green function (see, e.g., Ref. 9).

Integral equation (2) can be solved numerically in the case of a small number of defect rods. However, such calculations become severely restricted by the current computer facilities as soon as we increase the number of defect rods to create photonic-crystal waveguides, waveguide bends, and branches.[5–8] Therefore, our primary goal in this Letter is to develop a new approximate physical model that would allow the application of fast numerical techniques that combine reasonable accuracy and the further possibility of studying nonlinear photonic crystals and waveguides.

When the defects support monopole modes, a reasonably accurate model can be derived by the assumption that the electric field inside a defect rod remains constant. In this case, we can average the electric field in integral equation (2) over the cross section of the rods[9,12] and derive an approximate matrix equation for the amplitudes of the electric field, $E_n(\omega) \equiv E(\mathbf{x}_n|\omega)$, at the defect sites:

$$\sum_m M_{n,m}(\omega)E_m = 0,$$

$$M_{n,m}(\omega) = \epsilon_d J_{n,m}(\omega) - \delta_{n,m}\,, \tag{3}$$

where $\delta_{n,m}$ is Dirac's delta function and

$$J_{n,m}(\omega) = (\omega/c)^2 \int_{r_d} \mathrm{d}^2\mathbf{y} G(\mathbf{x}_n, \mathbf{x}_m + \mathbf{y}|\omega) \tag{4}$$

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is a coupling constant determined through the Green function of a perfect 2D photonic crystal.[9,10]

To check the accuracy of approximate model (3), first we consider a single defect located at point $\mathbf{x}_0$. In this case, Eq. (3) yields $J_{0,0}(\omega_d) = 1/\epsilon_d$, and this expression defines the frequency ω_d of the defect mode. For example, applying this approach to the case in which we have a defect created by a single removed rod, we obtain the frequency $\omega_d = 0.391 \times 2\pi c/a$, which differs by only 1% from the value $\omega_d = 0.387 \times 2\pi c/a$ calculated with the MIT Photonic-Bands numerical code.[3]

A single-mode waveguide can be created by removal of a row of rods (see the inset in Fig. 1). Assuming that the waveguide is straight ($M_{n,m} \equiv M_{n-m}$) and neglecting the coupling between separated defect rods (i.e., $M_{n-m} = 0$ for all $|n - m| > L$), we rewrite Eq. (3) in the transfer-matrix form: $\mathbf{F}_{n+1} = \hat{T}\mathbf{F}_n$, where we introduce vector $\mathbf{F}_n = \{E_n, E_{n-1}, \ldots, E_{n-2L+1}\}$ and transfer matrix $\hat{T} = \{T_{i,j}\}$ with the nonzero elements

$$T_{1,j}(\omega) = -\frac{M_{L-j}(\omega)}{M_L(\omega)}, \qquad j = 1, 2, \ldots, 2L,$$

$$T_{j,j+1} = 1, \qquad j = 1, 2, \ldots, 2L - 1. \tag{5}$$

Solving the eigenvalue problem

$$\hat{T}(\omega)\mathbf{\Phi}^p = \exp[ik_p(\omega)]\mathbf{\Phi}^p, \tag{6}$$

we can find the $2L$ eigenmodes of the photonic-crystal waveguide. The eigenmodes with real wave numbers $k_p(\omega)$ correspond to the propagating waveguide modes. In the waveguide shown in Fig. 1, there exist only two such modes (we denote them $\mathbf{\Phi}^1$ and $\mathbf{\Phi}^2$), propagating in opposite directions ($k_1 = -k_2 > 0$). In Fig. 1 we plot the dispersion relation $k_1(\omega)$ found from Eq. (6) for the nearest-neighbor interaction ($L = 1$) and also for the interaction among seven neighbors ($L = 7$); we compare the results with those calculated directly by the supercell method.[3] As soon as we go beyond the approximation of the nearest neighbors and take into account the coupling among several defect rods, Eqs. (3)–(6) provide very accurate results for the dispersion characteristics of the photonic-crystal waveguides. We verify that this conclusion is also valid for multimode waveguides, e.g., those created by removal of several rows of rods.

In addition to the propagating guided modes, in photonic-crystal waveguides there always exist evanescent modes with imaginary k_p. These modes, which cannot be accounted for in the framework of the nearest-neighbor approximation, remain somewhat "hidden" in straight waveguides, but they become important in more-elaborate structures such as waveguide bends and branches. Importantly, our model does take into account all such effects.

We consider the simplest case of a waveguide bend, in which the evanescent modes manifest themselves in two different ways. First, they create localized bound states in the vicinity of the bend. As was shown in Ref. 4, when the waveguide bend can be considered as a finite section of a waveguide of different type, the bound states correspond closely to cavity modes excited in this finite section. However, such a simplified one-dimensional model does not describe correctly more-complicated cases, even the bent waveguide depicted in Fig. 2. The situation becomes even more complicated for the waveguide branches.[7] In contrast, by solving Eq. (3) we can find the frequencies and profiles of the bound states that are excited in an arbitrary complex set of defects. As an example, in Fig. 2 we plot the profiles of two bound states (cf. Fig. 9 of Ref. 4). The frequencies of the modes are found from Eq. (3) with an accuracy of 1.5%.

Additionally, the evanescent modes determine the nontrivial transmission properties of the waveguide bends, which can also be calculated with our discrete equations. To demonstrate this, we consider a bent waveguide consisting of two coupled semi-infinite

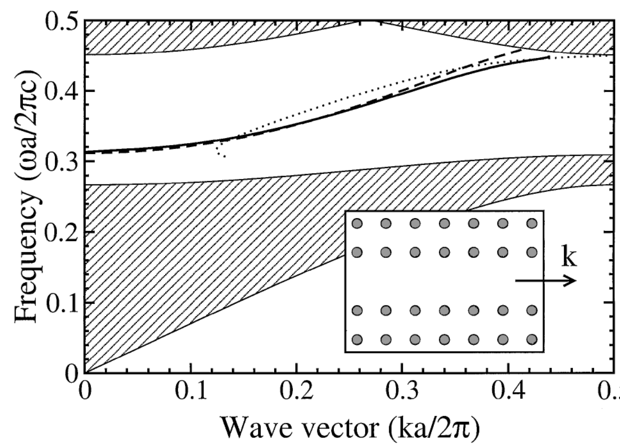

Fig. 1. Dispersion relation for the 2D photonic-crystal waveguide (shown in the inset) calculated by the supercell method[3] (dashed curve) and from approximate equations (5) and (6) for $L = 7$ (solid curve) and $L = 1$ (dotted curve). The hatched areas are the projected band structure of a perfect 2D crystal.

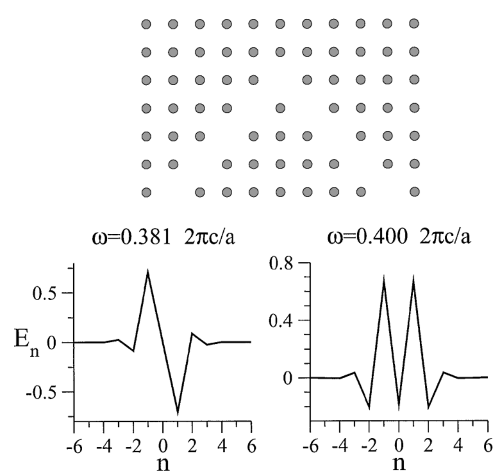

Fig. 2. Electric field E_n for two bound states supported by a 90° waveguide bend (shown at the top). The center of the bend is located at $n = 0$.

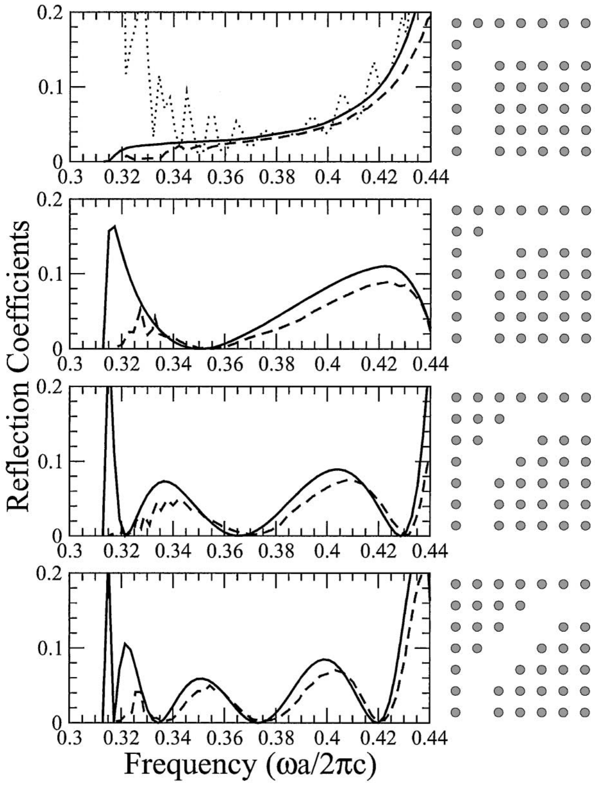

Fig. 3. Reflection coefficients calculated by the finite-difference time-domain method (dashed curves, from Ref. 5) and from Eqs. (3)–(6) with $L = 7$ (solid curves) and $L = 1$ (dotted curve in the top plot), for four different bend geometries.

straight waveguides with a finite section of defects between them. The finite section includes a bend with a safety margin of the straight waveguide at both ends. We assume that the defect rods inside this segment are characterized by the index that runs from a to b and that the amplitudes $E_m (m = a, \ldots, b)$ of the electric field near the sites of the removed rods are all unknown. We number the guided modes described by Eq. (6) in the following way: $p = 1$ corresponds to the mode propagating in the direction of the waveguide bend (for both ends of the waveguide); $p = 2$, to the mode propagating in the opposite direction; $p = 3, \ldots, L + 1$, to the evanescent modes that grow in the direction of the bend; and $p = L + 2, \ldots, 2L$, to the evanescent modes that decay in the direction of the bend. Then we can write the incoming and outcoming waves in the semi-infinite waveguide sections as a superposition of the guided modes:

$$E_m^{\rm in} = \Phi_{a-m}^1 + r\Phi_{a-m}^2 + \sum_{p=3}^{L+1} \lambda_p^{\rm in} \Phi_{a-m}^p \,, \tag{7}$$

for $m = a - 2L, \ldots, a - 1$, and

$$E_m^{\rm out} = t\Phi_{m-b}^2 + \sum_{p=3}^{L+1} \lambda_p^{\rm out} \Phi_{m-b}^p \tag{8}$$

for $m = b + 1, \ldots, b + 2L$, where $\lambda_p^{\rm in}$ and $\lambda_p^{\rm out}$ are unknown amplitudes of the evanescent modes growing in the direction of the bend and t and r are unknown amplitudes of the transmitted and reflected propagating waves. We take into account that the evanescent modes growing in the direction from the bend vanish and assume that the amplitude of the incoming plane wave Φ^1 is normalized to unity. Now, substituting Eqs. (7) and (8) into Eq. (3), we obtain a system of linear equations with $2L + b - a + 1$ unknown. Solving this system, we find the transmission $|t|^2$ and reflection $|r|^2$ coefficients.

In Fig. 3 we present our results for the transmission spectra of several types of bent waveguides, as in Ref. 5, in which the possibility of high transmission through sharp bends in photonic-crystal waveguides was demonstrated. As can be clearly seen, Eqs. (3)–(8) provide a very accurate method for calculating the transmission spectra of the waveguide bends.

In conclusion, we have suggested a novel conceptual approach for describing the properties of photonic-crystal waveguides and circuits, including the transmission spectra of sharp bends. The effective discrete equations that we have introduced here emphasize the important role of the evanescent modes in the photonic-crystal circuits, and one can apply them to study more-complicated problems such as transmission in waveguide branches, channel drop filters, and nonlinear localized modes in nonlinear waveguides.

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