Scattering matrix approach to large-scale photonic crystal circuits

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Received October 15, 2002

We propose a scattering matrix approach to the modeling of large-scale photonic crystal circuits and show that the transmission properties of complex circuits can be accurately calculated on the basis of scattering matrices of individual photonic crystal devices and waveguides that connect them. In addition, we show that functional devices such as waveguide bends generally exhibit a discontinuous frequency dependence in the phases associated with their complex reflection and transmission coefficients and emphasize its importance for the adequate modeling of photonic crystal circuits. © 2003 Optical Society of America

OCIS codes: 230.3120, 230.3990, 130.3120, 230.7370.

Over the past decade, substantial research efforts have been directed at investigations of photonic crystals (PCs) with embedded defects such as microcavities and waveguides. These structures hold tremendous potential for the creation of large-scale photonic integrated circuits, which are required for the next generation of high-throughput optical communications systems. Indeed, recent experimental attempts to fabricate the simplest types of PC-based optical circuits were very promising.[1]

Until now, the transmission properties[2] of PC-based circuits are mostly studied using the finite-difference time-domain method. This approach requires substantial computational resources and, as a consequence, modeling has been restricted to small-scale circuits. Another promising approach utilizes localized functions, such as localized defect modes[3,4] or photonic Wannier functions.[5] Such an approach combines accuracy with simplicity and provides intuitive insights into the properties of small-scale PC-based circuits.[3–5] In particular, PC waveguides support not only propagating guided modes but also a number of evanescent guided modes that are crucial for understanding the properties of waveguide bends.[3,4] However, even this approach is quite involved when one is designing large-scale PC-based circuits.

In this Letter we suggest a novel approach to fast and accurate designs of large-scale PC-based circuits. We show that such circuits may be considered a system of point-sized functional devices that are described by scattering matrices (S matrices) and are connected by PC waveguides with corresponding S matrices. The properties of the entire circuit are then accurately described by recursive combination of the individual S matrices of the functional devices and the waveguides into a total S matrix. It should be noted that there have been already suggestions to use S-matrix algorithms for modeling layered diffraction gratings[6] and small-scale integrated optics devices such as PC waveguides and air-bridge microcavities.[7] However, these algorithms are based on S matrices of tiny homogeneous slices that become computationally expensive for large-scale circuits.

To illustrate our approach, we consider PC-based circuits that consist of linear two-port devices, where we label any device (including waveguides) within this circuit by an index j. Furthermore, we denote the amplitudes of all guided modes (propagating and evanescent) between circuit elements $j-1$ and j by the p-component vectors $\mathbf{u}_j$ (modes propagating from device $j-1$ to device j) and $\mathbf{d}_j$ (modes propagating from device j to device $j-1$). Then, the behavior of device j is fully characterized by S-matrix $\hat{S}_j$, which connects the various incoming and outgoing modes according to

$$\begin{pmatrix} \mathbf{u}_{j+1} \\ \mathbf{d}_j \end{pmatrix} = \hat{S}_j \begin{pmatrix} \mathbf{u}_j \\ \mathbf{d}_{j+1} \end{pmatrix}, \qquad \hat{S}_j = \begin{bmatrix} \hat{T}_j^{uu} & \hat{R}_j^{ud} \\ \hat{R}_j^{du} & \hat{T}_j^{dd} \end{bmatrix}. \tag{1}$$

Here, the choice of $\hat{T}$ and $\hat{R}$ (transmission and reflection matrices) makes explicit the physical meaning of the four submatrices of $\hat{S}_j$. Generally, S matrices $\hat{S}_j$ should be calculated numerically, for instance, by the finite-difference time-domain method,[2] by use of grating-based S-matrix algorithms,[6,7] or from effective discrete equations.[3–5] However, for perfect nonabsorptive PC waveguides of length L and known dispersion relations $k_i(\omega)$ $i = 1, \ldots, p$ of the guided modes, the reflection matrices vanish, $\hat{R}_{\mathrm{wg}}^{ud} = \hat{R}_{\mathrm{wg}}^{du} \equiv 0$, and the transmission matrices, $\hat{T}_{\mathrm{wg}}^{uu}$ and $\hat{T}_{\mathrm{wg}}^{dd}$ are diagonal:

$$\hat{T}_{\mathrm{wg}}^{uu} = (\hat{T}_{\mathrm{wg}}^{dd})^* = \begin{bmatrix} \exp[ik_1(\omega)L] & 0 & 0 \\ 0 & \cdots & 0 \\ 0 & 0 & \exp[ik_p(\omega)L] \end{bmatrix}. \tag{2}$$

In the case of absorptive,[7] finite-height,[8] or disordered waveguides, their S matrices become more complex and should be calculated explicitly.

The S matrix of the entire circuit can then be calculated recursively from the S matrices of the individual functional elements that make up the circuit. For instance, S matrix $\hat{S}_c = \hat{S}_a * \hat{S}_b$ of a device c consisting of two subdevices, a and b, with S matrices $\hat{S}_a$ and $\hat{S}_b$, respectively, is given by[6]

$$\hat{T}_c^{uu} = \hat{T}_b^{uu}(1 - \hat{R}_a^{ud}\hat{R}_b^{du})^{-1}\hat{T}_a^{uu},$$
$$\hat{R}_c^{du} = \hat{R}_a^{du} + \hat{T}_a^{dd}\hat{R}_b^{du}(1 - \hat{R}_a^{ud}\hat{R}_b^{du})^{-1}\hat{T}_a^{uu},$$
$$\hat{R}_c^{ud} = \hat{R}_b^{ud} + \hat{T}_b^{uu}\hat{R}_a^{ud}(1 - \hat{R}_b^{du}\hat{R}_a^{ud})^{-1}\hat{T}_b^{dd},$$
$$\hat{T}_c^{dd} = \hat{T}_a^{dd}(1 - \hat{R}_b^{du}\hat{R}_a^{ud})^{-1}\hat{T}_b^{dd}. \quad (3)$$

The generalization of this S-matrix approach to multiport PC-based circuits such as waveguide branches and add–drop filters is straightforward. In addition, if the individual devices are connected through sufficiently long waveguides, a very convenient approximation consists of neglecting the guided evanescent modes.

As an example, we consider a two-dimensional PC consisting of a square array (with lattice spacing a) of infinitely long dielectric rods (with radius $r = 0.18a$ and dielectric constant $\epsilon = 11.56$) in air. For TM-polarized light propagating in the plane of periodicity, this PC exhibits a large photonic bandgap for frequencies from $\omega a/2\pi c = 0.303$ to $\omega a/2\pi c = 0.444$. By removing a row of rods, one can create a single-mode PC waveguide whose dispersion relation can be calculated with the supercell method. Waveguide bends can be constructed in a similar fashion and, by means of an appropriate design, light transmission through a bend can be optimized.[2]

In Fig. 1, we display the frequency dependence of the complex reflection coefficients, $R(\omega)$, for two different bend geometries. In particular, the reflection amplitude $\rho(\omega)$ of the roundish bend (b) vanishes at several resonance frequencies, and we want to emphasize that at exactly these resonance frequencies the phase $\phi(\omega)$ of the reflection coefficient experiences a nontrivial discontinuity. The complex transmission coefficients, $T(\omega)$, display an analogous behavior and, together with the reflection coefficients, $R(\omega)$, completely determine the bends' S matrix if we neglect the evanescent modes as discussed above.

We now consider a system of two bends connected by a waveguide of length L, which is currently under intense experimental investigation.[9] Using finite-difference time-domain calculations, it has been shown[10] that the transmission through a double-bend structure may be quite different from that through a single bend. However, to the best of our knowledge, no systematic investigation of this issue has been carried out to date. Within the S-matrix approach, the transmission properties of a double-bend structure follow directly from its S matrix, $\hat{S}(\omega) = \hat{S}_{\text{bend}}(\omega) * \hat{S}_{\text{wg}}(\omega, L) * \hat{S}_{\text{bend}}(\omega)$. In the present case we can explicitly evaluate the transmission coefficient in terms of the single bend and waveguide parameters as

$$|T|^2 = \frac{[1 - \rho^2(\omega)]^2}{1 + \rho^4(\omega) - 2\rho^2(\omega)\cos[2k(\omega)L + 2\phi(\omega)]}, \quad (4)$$

which coincides with the expression for transmission through a Fabry–Perot resonator with identical mirrors. In Figs. 2 and 3, we show the frequency dependence of the transmission T through double-bend structures constructed from the bends of Figs. 1(a) and 1(b), respectively. The results obtained from direct numerical simulations[3,4] virtually coincide with the results of Eq. (4) when only the single bend and the waveguide parameters are used. In fact, we find that this agreement is practically exact for waveguide lengths $L \geq 4a$. Furthermore, we want to emphasize the importance of the phase $\phi(\omega)$ in the complete characterization of a single waveguide. This is illustrated in Fig. 3, where we compare the results of Eq. (4) using the full phase information [solid curve;

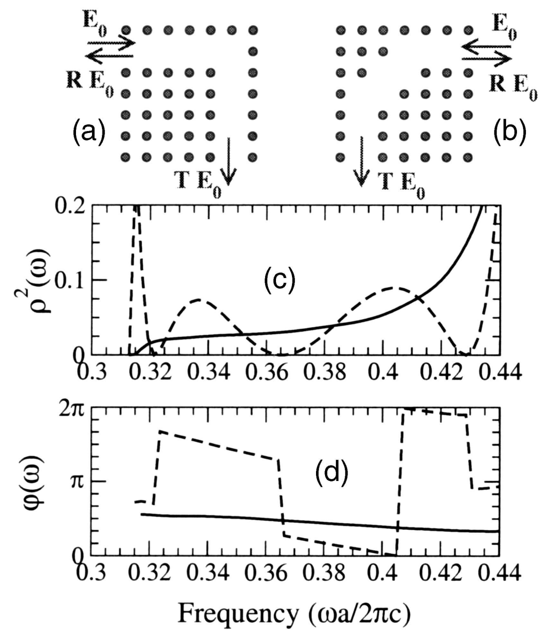

Fig. 1. (a), (b) Geometries of two different waveguide bends, whose complex reflection coefficients, $R(\omega) = \rho \exp(i\phi)$, are characterized by the (c) absolute value, $\rho(\omega)$, and (d) phase, $\phi(\omega)$, and have been calculated with effective discrete equations.[3] In (c) and (d), the solid lines correspond to the bend in (a) and the dashed curves, to the bend in (b).

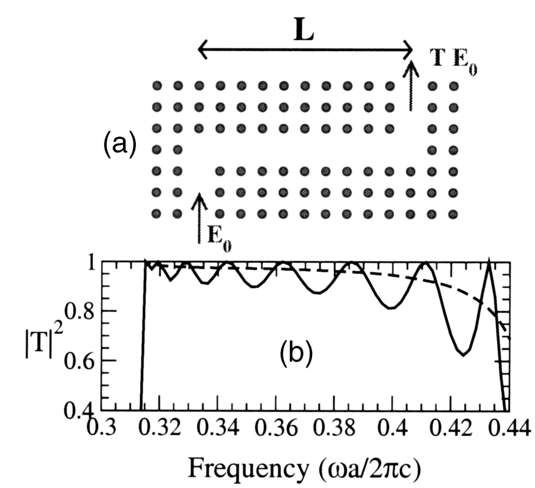

Fig. 2. (a) Double-bend system with bends of the type shown in Fig. 1(a) connected by a waveguide of length $L = 10a$ and (b) frequency dependence of the corresponding transmission T (solid curve). For reference, the transmission through a single bend is shown (dashed curve).

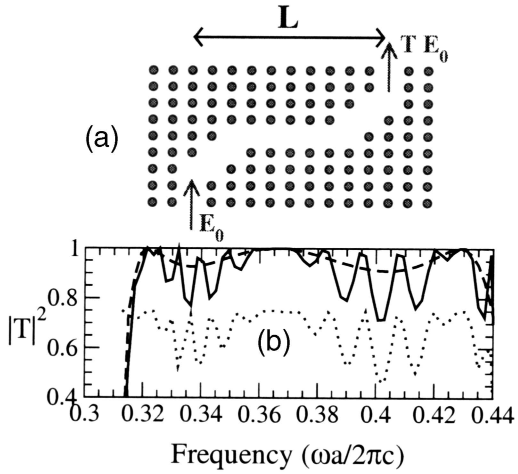

Fig. 3. (a) Double-bend system with bends of the type shown in Fig. 1(b) connected by a waveguide of length $L = 20a$ and (b) frequency dependence of the corresponding transmission T (solid curve). For reference, the transmission through a single bend is shown (dashed curve). The importance of the phase $\phi(\omega)$ [see Fig. 1(d)] is illustrated by evaluation of Eq. (4) for $\phi(\omega) = 0$ (dotted curve; values for T have been shifted downward by 0.25).

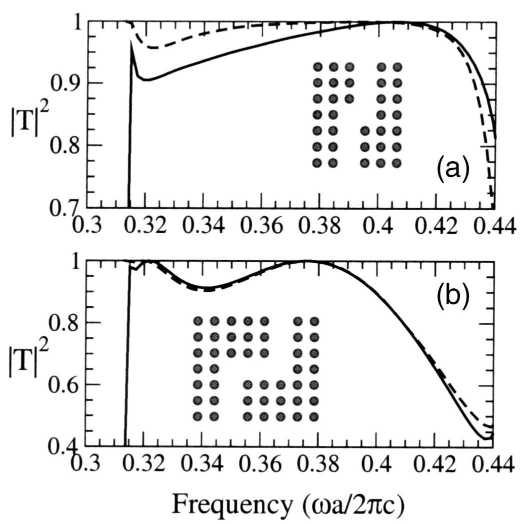

Fig. 4. Frequency dependence of the transmission T for double-bend structures similar to Fig. 2(a) with connecting waveguides of lengths (a) $L = a$ and (b) $L = 3a$. Results of direct numerical simulations (solid curves) and results of Eq. (4) (dashed curve) are shown.

see also Fig. 1(d)] with the results for a constant phase $\phi(\omega) = 0$. The inadequacy of disregarding the phase information is apparent.

To demonstrate the effect of neglecting guided evanescent modes, we compare in Fig. 4 the frequency dependence of transmission T from numerical simulations (solid curves) with the results of Eq. (4) (dashed curves) for the double-bend structure of Fig. 2 with reduced waveguide lengths, $L = a$ and $L = 3a$, respectively. When the bend separation is increased from $L = a$ to $L = 3a$, the evanescent mode coupling between the bends reduces substantially, and excellent agreement between numerical simulations and Eq. (4) is restored.

In conclusion, we have introduced an S-matrix approach to the optical properties of large-scale PC circuits. In this approach, individual functional elements such as bends and waveguides are characterized by an S-matrix, which allows efficient computation of the S matrix of the entire circuit from the knowledge of its constituent elements, not all of which have to be PC based. We have applied this approach to double-bend waveguiding structures and obtained excellent agreement with direct numerical simulations. Furthermore, we have discussed the importance of the phase of the reflection and transmission coefficients of the constituent functional elements and the conditions for neglecting guided evanescent modes. The results of our analysis are of direct relevance to the design of PC-based circuits as well as to the interpretation of experimental data. For instance, an analysis of the Fabry–Perot oscillations in the transmission of a waveguide sandwiched between two bends may allow the experimental determination of its dispersion relation (or lattice constant) through Eq. (4). Furthermore, the discrepancy between experimental results and Eq. (4) allows us to estimate the level of absorption and fabricational disorder within the PC waveguide.

We are indebted to Georg Bastian and Wolfgang Freude for illuminating discussions. This work was supported by the Center for Functional Nanostructures of the Deutsche Forschungsgemeinschaft. The research of K. Busch is further supported by DFG project Bu 1107/2-2 (Emmy-Noether program). S. F. Mingaleev's e-mail address is smino@tkm.physik.uni-karlsruhe.de.

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References

1. T. Krauss and T. Baba, eds., feature on Photonic Crystal Structures and Applications, IEEE J. Quantum Electron. **38**(7), 724 (2002).
2. A. Mekis, J. C. Chen, I. Kurland, S. H. Fan, P. R. Villeneuve, and J. D. Joannopoulos, Phys. Rev. Lett. **77,** 3787 (1996).
3. S. F. Mingaleev and Yu. S. Kivshar, Opt. Lett. **27,** 231 (2002).
4. S. F. Mingaleev and Yu. S. Kivshar, J. Opt. Soc. Am. B **19,** 2241 (2002).
5. A. Garcia-Martin, D. Hermann, K. Busch, and P. Wölfle, Mater. Res. Soc. Proc. **722,** L1.1 (2002).
6. L. Li, J. Opt. Soc. Am. A **13,** 1024 (1996).
7. E. Silberstein, Ph. Lalanne, J.-P. Hugonin, and Q. Cao, J. Opt. Soc. Am. A **18,** 2865 (2001).
8. M. Palamaru and Ph. Lalanne, Appl. Phys. Lett. **78,** 1466 (2001).
9. E. Chow, S. Y. Lin, J. R. Wendt, S. G. Johnson, and J. D. Joannopoulos, Opt. Lett. **26,** 286 (2001).
10. A. Chutinan and S. Noda, Phys. Rev. B **62,** 4488 (2000).