Time-and-Frequency-Domain Modeling (TFDM) of Hybrid Photonic Integrated Circuits
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ABSTRACT
This work addresses the efficient modeling of hybrid large-scale photonic integrated circuits (PICs) comprising both, active and passive sub-elements. We describe a new modeling approach, the time-and-frequency-domain modeling (TFDM) that improves accuracy, memory requirements and simulation speed in comparison with traditional pure time-domain method. In TFDM, clusters of connected linear PIC elements are modeled in frequency domain, while interconnections between such clusters and non-passive PIC elements are modeled in the time domain. Behavioral models of the fundamental building blocks of PICs are presented and combined in several application examples showing the robustness of the entire modeling framework for PICs.

Keywords: photonic integrated circuit, scattering matrix, design, modeling, photonic integration, time-and-frequency domain modeling

1. INTRODUCTION
The competence of integrating a large number of optical elements on single wafer-dices involves the design of complex photonic chips that might contain hundreds of elements with diverse functionalities. Photonic Design automation (PDA) can contribute to the progress of this by providing compliant, time efficient and reliable environments for the simulation of photonic integrated circuits (PICs).

In previous works [1,2] we have addressed the modeling of fully passive PICs, based on the description of PIC elements in terms of frequency-dependent scattering-matrices [3]. Such photonic circuits consist of linear sub-elements, as is the case of most of the Silicon-based PICs, and can be efficiently analyzed in the frequency domain. However, in a more general situation, photonic chips contain non-linear or active elements as well, requiring a time-domain description.

In approaches up to date, even in the presence of only a single non-passive element, this type of hybrid circuits has been modeled in the time-domain. In doing so, FIR filters have been calculated for each S-matrix describing a passive element in time-domain. Although FIR design methods provide a very accurate solution at the center of the signal band, their accuracy intrinsically degrades near the band edges. This turns into a problem in large-scale integrated circuits, due to multiplicative effects in the inaccuracy that makes it a non-scalable solution for modeling hybrid PICs.

Quite recently, we elaborated [4] an alternative time-and-frequency domain modeling (TFDM) approach that allows to overcome the above outlined limitations of the pure time-domain simulations and enables efficient and scalable modeling of hybrid large-scale PICs. With this contribution, we present our implementation of the TFDM approach and demonstrate its efficiency for several application examples.
2. SYSTEM-LEVEL ABSTRACTION OF PHOTONIC CIRCUITS

Traditional simulators of photonic devices are typically based on the solution of the Maxwell equations of the entire structure (for example, the Finite-Difference Time-Domain method and the Beam-Propagation method). The computational requirements of simulation time and resources when directly solving the Maxwell equations grows more than proportionally to the number of PIC sub-elements. To deal with novel large-scale integrated circuits would require the use of a computer cluster for modeling even medium-scale PICs, and might be unfeasible for design applications that consist of many hundreds of basic photonic elements.

![Figure 1: System-level modeling approach implemented in VPIcomponentMaker Photonic Circuits](image)

To cover this problem, we introduced an abstracted system-level modeling technique [5] that consists of the segmentation of the PIC into basic building blocks, named PIC elements (Figure 1). Each PIC element is considered as a black box, characterized by several connecting ports. A PIC design might comprise both, active and passive devices, which can be modeled by different methods and with different level of simulation accuracy. Passive PIC elements are described in terms of frequency-defined scattering matrices (S-matrices), as they don't require a deep level of detail compared to active sub-devices, such as semiconductor lasers, amplifiers or absorbers, which are modeled by means of the transmission-line model (TLM). Furthermore, the segmentation method allows to separate system-level modeling from device-level modeling. Thus, the device modeling can be performed either using detailed traditional photonic simulators or employing analytical and behavioral models of the PIC elements.

It can be foreseen that passive circuits are efficiently modeled in the frequency domain, and PIC designs comprising active, non-linear or dynamically-tunable devices, involve more detailed simulations in time-domain. However, neither pure frequency- nor time-domain simulations offer the best solution to cover the variety of upcoming novel devices. In contrast to traditional simulation tools, which implement either frequency-domain or time-domain simulation approaches, our modeling approach supports both, and can be also combined within a single simulation.

2.1. Frequency-domain modeling of linear passive circuits

The frequency-domain approach is the most efficient and accurate way to simulate purely passive PICs that contain only linear elements. In frequency-domain simulations, signals are processed by modules block-wise so that each module waits until the input signals are known and then processes them and obtains output signals which are passed to the next module. This process is usually very fast and can be effectively parallelized in setups with parallel signal propagation paths. Output signals are calculated in this domain according to expression (1), denoting the spectra of input and output signals at port \( n \) as \( E_{n}^{(in)}(f) \) and \( E_{n}^{(out)}(f) \).

\[
\begin{pmatrix}
E_{1}^{out} \\
E_{2}^{out} \\
\vdots \\
E_{N}^{out}
\end{pmatrix}
\begin{pmatrix}
\hat{T}_{1}^{in} \\
\hat{T}_{2}^{in} \\
\vdots \\
\hat{T}_{N}^{in}
\end{pmatrix}
= \hat{S}(f)
\begin{pmatrix}
E_{1}^{in} \\
E_{2}^{in} \\
\vdots \\
E_{N}^{in}
\end{pmatrix}
\tag{1}
\]

where \( \hat{S}(f) \) is the S-matrix describing the PIC element. The S-matrix consists of \( N \times N \) Jones matrices \( \hat{T}_{nm}(f) \) connecting the input port \( m \) with the output port \( n \). In its turn, each Jones matrix is formed by four complex-valued frequency-dependent transfer functions: \( T_{nm}^{TE,TE}(f) \), \( T_{nm}^{TE,TM}(f) \), \( T_{nm}^{TM,TE}(f) \), and \( T_{nm}^{TM,TM}(f) \). The different transfer functions can be calculated analytically or loaded from data files.
Representative examples of purely passive PICs are arrayed waveguide gratings (AWG). An AWG is typically built from two star couplers and several hundreds of optical waveguides. In this example (Figure 2), a mapping module is used for generating copies of the optical waveguide element with different phase shifts added randomly. The length of two subsets of waveguides is increased with different length increments, resulting in a flat channel pass-band WDM filter.

Figure 2. Example of frequency-domain modeling of an AWG; Simulation setup (top) and transmission spectra (bottom)

2.2. S-Matrix domain of linear passive circuits
This modeling technique can be successfully used in numerous applications. However, in some situations, we can face several peculiarities. One example is when circuits enclose feedback loops between the PIC elements. This is illustrated in the following example (Figure 3). The PIC that is evaluated here consists of two coupled silicon ring resonators demonstrating the phenomenon known as coupled-resonator-induced transparency (CRIT) [6].

Simulation of such a schematic would require multiple iterations for the output data to converge. The number of iterations would depend on the quality factor of the resonances; structures with high-Q resonances might require
hundreds of iterations. Here, the special modules 'repeat' and 'chop' assist the recurring simulations so that transient dynamics are skipped and only the last iteration is passed to the data analysis modules. Additionally, logical delays between PIC sub-elements are required to prevent a simulation deadlock. Altogether the effort becomes tiresome, even for this rather simple example with only two ring couplers and two waveguides.

Alternatively, these steps can be avoided by assembling S-matrices of individual PIC elements into the S-matrix of the whole passive device (Figure 4). In contrast to the previous solution, fully-converged results are achieved with only a single iteration (against 160 in the previous simulation approach), because the total S-matrix is preliminary calculated before signal processing. Consequently, the same simulation can be performed much faster, and the setup is more simple (no need to use repeat and chop elements, neither logical delays).

2.3. Time-domain modeling of passive circuits

As already discussed, pure frequency-domain simulations are not appropriate in structures comprising non-passive PIC elements, such as lasers, SOAs, and modulators that require a sample-by-sample based time-domain procedure. This implies that all passive PIC elements should be operated also in the time domain requiring that output signals are calculated by means of the convolution between the input signals and the device impulse response. The impulse response of a linear passive element is calculated by means of FIR filters, designed on the basis of their corresponding S-matrix. The elaboration of such FIR filters with an acceptable accuracy represents one of the major modeling tasks.

Even though high precision designs are realizable there is always intrinsic deterioration near the edges of the simulated signal bands. This is even noticeable in the case of a simple ring resonator (Figure 5). The response by using a FIR-based time domain model is quite accurate inside a large frequency range. However, it degrades severely at the edges of the simulation frequency window. Such inaccuracy is not representative in the case of having a few elements, but the total bandwidth of accurate simulations (for a given accuracy tolerance) decreases exponentially rapidly as the number of PIC elements in the simulated PIC increases.

![Figure 4: Representation of the CRIT using a single element composed by the total matrix of the single parts](image)

![Figure 5: Example of a ring-resonator model, comparing the approximated time-domain approach with the exact solution in a frequency-domain simulation](image)
This reduction of accurate simulation bandwidth is aggravated (for a given time step) in the presence of short-length linear PIC elements which are characterized by small group delays (for example, micro-rings with small diameters or short pieces of waveguides connecting other neighboring devices). An additional serious intensifier of even small simulation inaccuracies is attributed to feedback loops, always present in large-scale PICs. All this enforces, for keeping a prerequisite simulation bandwidth, to use smaller and smaller time steps as the complexity of the modeled PIC grows, thus precluding scalability of the described time-domain approach.

To illustrate the problems of purely time-domain modeling of PICs, let us come back to the system exhibiting the CRIT effect (see Figure 3) and try to model it in time domain (see Figure 6). To make this example more realistic, we have added to the modeled structure four 300 μm length access waveguides. Here, the short 5.83 μm length waveguides that connect two ring couplers introduce a small group delay of about 81 fs. Accurate modeling of such waveguides with FIR filters requires time steps which are at least 16 times smaller, i.e. about 5 fs. In other words, accurate modeling of such short waveguides in time domain requires usage of signals with an extreme wide bandwidth of about 200 THz. On the other hand, we would wish to resolve in our simulations the narrow coupled-resonator-induced transparency line at the center of the optical spectrum in Figure 3. This line with the quality factor $Q \approx 65300$ has a 3 dB passband around 3 GHz, and thus can be resolved only when the signal duration is equal or exceeds 600 ps (which corresponds to the usage of 120000 time samples). All this leads to unacceptably long simulation time of the CRIT structure in time domain (around 4 minutes on a personal computer with 3 GHz CPU, being 120 times slower than 2 seconds required for the modeling of the same structure with the frequency-domain S-matrix assembly technique illustrated in Figure 4). Moreover, using a signal bandwidth of approximately 200 THz means that this simulation time shall dramatically increase as soon as the modeled structure shall include some active PIC elements.

2.4. Time-and-frequency domain modeling (TFDM), our suggestion for scalable simulation technique

To overcome the limitations highlighted before and taking the advantage of frequency-domain simulations with assembled S-matrices, we have proposed [4] a new hybrid time-and-frequency domain modeling (TFDM) approach. Within this approach, clusters of interconnected passive PIC elements are identified as illustrated in Figure 6, and represented as a single element modeled by a single equivalent S-matrix. Correspondingly, the digital FIR filters will be generated for only these few passive clusters instead of hundreds of PIC elements, greatly improving simulation accuracy and speed.
To illustrate advantages of such our TFDM approach, let us consider the system exhibiting the CRIT effect once again (see Figure 7). Here, the FIR filters should be designed for the whole modeled device. The minimal group delay accumulated by the optical signal propagating inside this device is approximately 8.8 ps. Therefore, accurate modeling of such a device with FIR filters requires to use a time step of approximately 0.5 ps. Note that this is 100 times longer than the time step required for the pure time-domain approach!. Correspondingly, the required signal bandwidth is approximately 2 THz. The requirements for the signal duration remain unchanged, so that the total number of time samples is reduced also 100 times. Although the reduction in simulation time is not so dramatic for the purely passive structure (modeling of this device with the TFDM approach takes about 18 seconds on the same personal computer), it becomes very impressive as soon as the modeled structure shall include some active PIC elements (since the signal bandwidth of 2 THz allows to model active devices in only one to few minutes).

As an application of the TFDM technique to the modeling of a hybrid PIC containing active and passive elements, we present an example of wavelength conversion based on a SOA integrated with a multi-ring resonator. Multi-ring resonators with several microns ring radii allow for compact channel add-drop filters with flat pass-bands and a wide free spectral range (FSR) of about 30 nm [7]. Because of this, they are very attractive for various applications, including efficient separation of four-wave mixing terms from the pump and probe signals in compact wavelength-conversion devices [8]. However, due to small ring radii, the standard approach to model such resonators in time-domain requires usage of a huge, that is over 150 THz signal bandwidth which is incompatible with the efficient time-domain modeling of SOAs and other active devices.
The schematic shown in Figure 7, illustrates the combination of time- and frequency-domain models. Probe and pump signals are coupled at a waveguide coupler, which is described by an FIR filter. The three-ring filter is modeled also by an FIR filter constructed by assembling $S$-matrices of each PIC element. After this pre-calculation, a time-domain simulation is carried out. Results after simulation are shown in Figure 8. The spectrum after the SOA has several FWM terms which are efficiently attenuated after the ring filter.
3. SUMMARY

With this work, we have presented the recently developed simulation framework for photonic integrated circuits in the context of the professional design suite VPIcomponentMaker Photonic Circuits. We have shown that when each of the PIC elements comprising a photonic circuit is modeled by its own set of digital FIR filters, the modeling suffers from several principal limitations. Although the accuracy provided by FIR filters could always be improved, the accuracy in simulation deteriorates in the presence of a large number of elements and also in short-length devices and feedback loops. We stressed the necessity of combining several system-level modeling techniques within the same simulation setup in order to model realistic large-scale PICs efficiently. This can be accomplished within our demonstrated hybrid time-and-frequency-domain modeling (TFDM) approach. TFDM consists of first, calculating S-matrices of fully-linear PIC sub-clusters (with the S-matrix assembly technique) and then, designing the digital FIR filters for such sub-clusters instead of individual PIC elements.

4. REFERENCES